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Finite Element Method and 2d Wave Equation

The paper discusses the basic theoretical relationship of Finite Element Method and 2d Wave Equation. Wave equation has close relationship with finite element method as It is also used in other areas of theoretical physics, for example, in the description of gravitational waves. It is one of the basic equations of mathematical physics. This method is irreplaceable if it is necessary to take into account the geometric features of the regions — then the computer is used not only to solve a system of equations, but primarily to formulate and construct discrete approximations.

In the last five decades it has intensely researched the application of different numerical methods for the propagation of seismic waves. The main motivation for this is the fact that there are no analytical solutions for geophysical models of interest, for example, models with a distribution arbitrary heterogeneity or with topography. A partial differential equation is an equation which involves functions and their partial derivatives. For example, the average quantities like density, velocity, temperature, pressure, concentration or electromagnetic field are all determined using Partial Differential Equations (PDE). For many engineering and scientific purposes these equations represent as a language. Partial Differential equations are not as easy to solve as ordinary differential equations.

These local properties of propagation, which are extremely important in this type of problem, can not be studied by means of the global error estimation techniques, common in finite elements. A method is used here based on the decomposition of the continuous solution of the problem in its Fourier harmonics, and in the analysis of the propagation properties of each of these harmonics in the finite element mesh. Additionally, these same local properties of propagation of the different finite element meshes are studied in the environment of the artificial borders in which "transparent" boundary conditions of the first order are imposed. This study reveals the existence of partial phenomena in the artificial boundary (Foreman).

The basic finite-difference relations use simple schemes. calculating derivatives. Wave simulation with high accuracy for large time spacing requires very fine discretization in space and time. Consequently, increases the required amount of computer memory and its performance. In the calculations derivatives you can apply more complex finite-difference relations that have increased accuracy. Then we get an algorithm that uses rationally performance and computer memory. This approach was investigated in for classical one-dimensional wave equation.

The equations of the finite element method are also known as the theory of elasticity. The method of finite differences, widely used for solving plane issues of the theory of elasticity, becomes rather cumbersome in the case of regions with a complex contour. The rapidly developing finite element method, although it can be extended to spatial objects, is not without drawbacks, since it is associated with solving systems of high order algebraic equations. To a considerable degree of the noted deficiencies, the method of expanding a given system is deprived, but it still does not receive due attention (Ainsworth, 5-40).

There are also second order Partial differential equations (PDE) which are usually used in physics to represents shapes like parabolic, hyperbolic, and elliptic1. Parabolic equations explain the evolutionary process that guides to a firm state which was described by an elliptic equation. The elliptic equation shows an important state of a physical system which is identified through the lowest level of the certain quantity. Hyperbolic represents the transport of some physical quantity and information. There is also another type of second-order PDEs, but they are said to be undetermined.

A solution of the PDE is a function of u(x,y) of two independent variables that own all partial derivatives taking place in an equation and satisfies the equation some region of the XY-plane. Laplace equation represents the Elliptic partial differential equation, the Heat equation represents a parabolic partial differential equation, and the Wave equation represents the Hyperbolic partial differential equation.

The values and distributions of nominal stresses are initial for determining local stresses (mechanical and thermal) in places of constructive stress concentration (grooves, fillets, holes, threads, etc.). Local stresses can be estimated based on extensive reference information. According to the theoretical stress concentration coefficients obtained from the solution of the boundary problems of the theory of elasticity, as well as from experiments (in particular, by the method of photo elasticity). Significant opportunities in determining local stresses in concentration zones are associated with the expanding use of computers and numerical methods for solving boundary value problems (finite element methods, finite differences, boundary integral equations). In a large number of cases, local stresses in the concentration zones (taking into account temperature and residual stresses) can exceed yield strength, causing repeated elastoplastic deformation (Thompson 337-340).

A different formulation of the finite element method is possible, which follows from the idea that for any exact or approximate method for solving the problem of the theory of elasticity, equilibrium equations and compatibility conditions must be satisfied. In the displacement method described above, the distribution of displacements is assumed such that their consistency is ensured, therefore, with an approximate solution of the equilibrium equation, they are not exactly satisfied.

To conclude, there is a close relationship between wave equation with finite element method. Method finite elements is surprisingly successfully applied in a wide variety of problems with relation to wave equation. It was created to solve complex equations of elasticity theory and structural mechanics and turned out to be much more efficient than the finite-element method.

Works Cited

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