Linear Programming

Name of Student

[Institutional Affiliation(s)]

Author Note

The concept of linear programming was developed during the Second World War when there was a need to maximize the efficiency of available resources. Programming was a term used by military officials and included planning in way that used minimal resources. The simplex method was developed by a person from the U.S. Air force for solving problems which were linear in nature. A linear program contains an objective function that needs to be minimized or maximized according to given problem. There are decision variables available in the program as well. In our example, there is no specific objective function available but we can assume these values for a start. There are two variables in our example namely x1 and x2 which will represent the mower housing and tractor housing respectively. A set of variables which provides feasible solution to linear programming problem is called feasible region. All variables defined must fall within the feasible region otherwise one or more constraints will not be satisfied.

A linear programming problem can be defined as a situation where maximum or minimum values have to be found keeping some constraints in view which are also assumed to be linear. Constraints may be in the form of equation an inequality. In the problem under consideration, there are two unknown variables and 6 constraints. We have assumed that the values of both our variables will be positive. We have only placed the main constraints. The function which has to be maximized or minimized is called an objective function (Denardo, 2011).

There are several assumptions which have to be fulfilled before we can run a linear program on any data. Proportionality is described as a fact that the contribution of any variable to objective function moves with that variable. The contribution towards any objective function z, is proportional to the level of that activity cx. The contribution of each activity is also proportional to its level ax. In the objective function, proportionality shows that there is an equal addition to the contribution when we increase a unit any variable. This marginal rate of contribution will remain the same for the full range of activity. If we draw a diagram that shows contribution of a variable x1 to objective function z, it will form a straight line showing proportional relationship.

Additivity shows that every function is the sum of single contribution of defined activities. Thus,

Z = ∑cx

∑ ax ≤ b

The variables are assumed to be added or subtracted together and are never multiplied or divided by each other. In the objective function, this implies that contribution of variables towards the objective function is the total of their weighted contributions. The total profit in this case will be the total of individual profits earned by the products. If a business is managed in a proper manner, this assumption is also automatically fulfilled. In terms of constraints, this assumption shows that total usage of resources is the sum of amounts of resources used per variable which is normally the case.

There are no economies of scale assumed in the analysis because they will decrease the marginal addition rate in case of activities. Similarly, there will be no discounts provided to the customers because they will also violate the assumption of proportionality. If there is an ample representation of average marginal rate of contribution by the coefficients, this assumption is assumed to hold itself. In the constraints this assumption shows that resource usage by each variable is constant. In most production systems, this assumption holds by itself so it is reasonable to state that it is satisfied.

An example can be given that 8x1 will be double of 4x1. The contribution of variables to objective function is independent of each other. The values of variables can come out as fractions which show the assumption of divisibility. A line is a geometric identity and it can be formed by joining points which contain fractions. In the practical examples, we will often come across values of solution which will not be integers. The production is assumed to be a continuous process and this assumption is not a big obstacle in the calculations (G.V.Shenoy, 1998).

The assumption of certainty states that all variables are known with certainty. However, there is considerable guess work involved in arriving at variable values. There are no interceptions of probabilities in the model. This is an unreal assumption in the real world applications especially when the time period involved in longer and there are probabilistic methods which can be applied in this case.

A sensitivity analysis helps to determine the impact of changes in different variables. There can be many optimal solutions to a linear programming problem. The solutions may share some boundaries with each other and for any two solutions, they are said to be adjacent if they share n-1 boundaries of constraints. If a solution does not have any adjacent solutions, then it has to be an optimal solution.

The assumption of linearity is the sum of additivity and proportionality which implies that no variable is dependent upon any other variable. The resources are assumed to have only one particular usage over time and they cannot be substitutes or compliments to each other. In the practical examples, it will be difficult to apply the assumption of linearity. The violation of assumptions will mean that the situation will require applying some non linear programming techniques.

There are several cases in which linear programming problems do not form an exact plot or box. There is an option of presenting the variables in form of strict equalities which will be done by adding more variables to the statements. These variables will have a value equal to difference between left and right hand sides of original inequality. This process will be suitable for instances with less than or equal to constraints whereas for more than or equal to constraints, a variable is subtracted on left hand side of inequality.

If a variable can have a negative value in context of a specific problem, it is called an unrestricted in sign variable, we can replace the variable with difference of two values to make sure that it sets in box model. A sensitivity analysis deals with the impact of change in any one parameter on solution of model. The main problem in this method is that the values have to be in a canonical form which can be considered quite limited in nature. Setting any variable equal to zero will provide a solution for the other variable immediately. We have two basic variables but if there are more variables added then they can be set to zero to obtain the feasible values of basic variables. There are also situations where it will seem that increasing a variable will be feasible but there will be a single optimal value for the whole model. If, in a maximization situation, if some variable has a positive value in the objective as well as some constraint functions, there are more than one feasible solutions to the problem at hand that can be found by pivoting. When there is a new variable added to an analysis, there should be a minimum ratio of the right hand side value to the new variable added.

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| **Number to make** | 1933.33 | 100 |  |  |  |
|  |  |  |  |  |  |
| **Required Inputs** | |  |  |  |  |
| **Stamping** | 0.03 | 0.07 | 65 | ≤ | 200 |
| **Drilling** | 0.09 | 0.06 | 180 | ≤ | 300 |
| **Assembly** | 0.15 | 0.1 | 300 | ≤ | 300 |
| **Painting** | 0.04 | 0.06 | 83.3333 | ≤ | 220 |
| **Packaging** | 0.02 | 0.04 | 42.6667 | ≤ | 100 |
| **Sheet** | 1.2 | 1.8 | 2500 | ≤ | 2500 |
|  |  |  |  |  |  |
| **Objective** | 10 | 10 |  |  | 20333.3 |

The above table shows the original output from the linear programming output. The objective function is assumed to be 10 houses of each type and we see that the answers are not realistic in terms of number of houses so we will make some changes in the model to make sure that there is a more realistic answer to the problem. We will change the value of variables one by one in all constraints to see the impact of these changes. There will be some changes which will not set in the original constraints set by the model. There is no effect on the profit maximizing output when we change the value of a variable in the constraint because the conditions are fulfilled by a variety of values within the favourable region.

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| **Number to make** | 1933.33 | 100 |  |  |  |
|  |  |  |  |  |  |
| **Required Inputs** | |  |  |  |  |
| **Stamping** | 0.09 | 0.1 | 184 | ≤ | 200 |
| **Drilling** | 0.09 | 0.06 | 180 | ≤ | 300 |
| **Assembly** | 0.15 | 0.1 | 300 | ≤ | 300 |
| **Painting** | 0.04 | 0.06 | 83.3333 | ≤ | 220 |
| **Packaging** | 0.02 | 0.04 | 42.6667 | ≤ | 100 |
| **Sheet** | 1.2 | 1.8 | 2500 | ≤ | 2500 |
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| **Objective** | 10 | 10 |  |  | 20333.3 |

The above table shows upper limit of variable x1 and x2 which can be used to utilize the stamping hours within the given range. If we change the value of x1 or x2 above this level, it will not satisfy the constraint put in place. There is no effect on the profit maximizing output when we change the value of a variable in the constraint because the conditions are fulfilled by a variety of values within the favourable region.

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| **Number to make** | 1933.33 | 100 |  |  |  |
|  |  |  |  |  |  |
| **Required Inputs** | |  |  |  |  |
| **Stamping** | 0.09 | 0.1 | 184 | ≤ | 200 |
| **Drilling** | 0.15 | 0.06 | 296 | ≤ | 300 |
| **Assembly** | 0.15 | 0.1 | 300 | ≤ | 300 |
| **Painting** | 0.04 | 0.06 | 83.3333 | ≤ | 220 |
| **Packaging** | 0.02 | 0.04 | 42.6667 | ≤ | 100 |
| **Sheet** | 1.2 | 1.8 | 2500 | ≤ | 2500 |
|  |  |  |  |  |  |
| **Objective** | 10 | 10 |  |  | 20333.3 |

In the above table, we have changed the drilling variable of production and time taken to complete a mower house. Any further increase in the variable x1 in terms of drilling will break the constraint applied to the model. As far as the assembly variable is concerned, there is an exact value of the variable which has been calculated to be equal to the constraint when we alter value of x1.

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| **Number to make** | 1933.33 | 100 |  |  |  |
|  |  |  |  |  |  |
| **Required Inputs** | |  |  |  |  |
| **Stamping** | 0.09 | 0.1 | 184 | ≤ | 200 |
| **Drilling** | 0.15 | 0.06 | 296 | ≤ | 300 |
| **Assembly** | 0.15 | 0.1 | 300 | ≤ | 300 |
| **Painting** | 0.1 | 0.06 | 199.333 | ≤ | 220 |
| **Packaging** | 0.02 | 0.04 | 42.6667 | ≤ | 100 |
| **Sheet** | 1.2 | 1.8 | 2500 | ≤ | 2500 |
|  |  |  |  |  |  |
| **Objective** | 10 | 10 |  |  | 20333.3 |

The above table shows the changes in painting variable which have to be done to reach the value of constraint. There is no effect on the profit maximizing output when we change the value of a variable in the constraint because the conditions are fulfilled by a variety of values within the favourable region.

We have seen that there is a margin for increasing the number of hours utilized by each department keeping the constraints in place which means that the company is under utilizing the resources. In order to have further analysis, we must have an objective function at hand and solve it according to constraints. Similar changes can be predicted if we change the values of the variable x2.

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| **Number to make** | 1933.33 | 100 |  |  |  |
|  |  |  |  |  |  |
| **Required Inputs** | |  |  |  |  |
| **Stamping** | 0.03 | 1.4 | 198 | ≤ | 200 |
| **Drilling** | 0.09 | 0.06 | 180 | ≤ | 300 |
| **Assembly** | 0.15 | 0.1 | 300 | ≤ | 300 |
| **Painting** | 0.04 | 0.06 | 83.3333 | ≤ | 220 |
| **Packaging** | 0.02 | 0.04 | 42.6667 | ≤ | 100 |
| **Sheet** | 1.2 | 1.8 | 2500 | ≤ | 2500 |
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| **Objective** | 10 | 10 |  |  | 20333.3 |

In the above table, we have changed the value of variable x2 to see its maximum value for satisfying the constraint condition for stamping department.

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| **Number to make** | 1933.33 | 100 |  |  |  |
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| **Required Inputs** | |  |  |  |  |
| **Stamping** | 0.03 | 1.4 | 198 | ≤ | 200 |
| **Drilling** | 0.09 | 1.2 | 294 | ≤ | 300 |
| **Assembly** | 0.15 | 0.1 | 300 | ≤ | 300 |
| **Painting** | 0.04 | 0.06 | 83.3333 | ≤ | 220 |
| **Packaging** | 0.02 | 0.04 | 42.6667 | ≤ | 100 |
| **Sheet** | 1.2 | 1.8 | 2500 | ≤ | 2500 |
|  |  |  |  |  |  |
| **Objective** | 10 | 10 |  |  | 20333.3 |

In the above table, we have changed the value of variable x2 to see its maximum value for satisfying the constraint condition for drilling department. The shown values also satisfy the constraint condition for assembly department as well. There is no effect on the profit maximizing output when we change the value of a variable in the constraint because the conditions are fulfilled by a variety of values within the favourable region.

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| **Number to make** | 1933.33 | 100 |  |  |  |
|  |  |  |  |  |  |
| **Required Inputs** | |  |  |  |  |
| **Stamping** | 0.03 | 1.4 | 198 | ≤ | 200 |
| **Drilling** | 0.09 | 1.2 | 294 | ≤ | 300 |
| **Assembly** | 0.15 | 0.1 | 300 | ≤ | 300 |
| **Painting** | 0.04 | 1.4 | 217.333 | ≤ | 220 |
| **Packaging** | 0.02 | 0.04 | 42.6667 | ≤ | 100 |
| **Sheet** | 1.2 | 1.8 | 2500 | ≤ | 2500 |
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| **Objective** | 10 | 10 |  |  | 20333.3 |

In the above table, we have shown the maximum value of the variable x 2 which can be used to satisfy constraint in painting department.

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| **Number to make** | 1933.33 | 100 |  |  |  |
|  |  |  |  |  |  |
| **Required Inputs** | |  |  |  |  |
| **Stamping** | 0.03 | 1.4 | 198 | ≤ | 200 |
| **Drilling** | 0.09 | 1.2 | 294 | ≤ | 300 |
| **Assembly** | 0.15 | 0.1 | 300 | ≤ | 300 |
| **Painting** | 0.04 | 1.4 | 217.333 | ≤ | 220 |
| **Packaging** | 0.02 | 0.6 | 98.6667 | ≤ | 100 |
| **Sheet** | 1.2 | 1.8 | 2500 | ≤ | 2500 |
|  |  |  |  |  |  |
| **Objective** | 10 | 10 |  |  | 20333.3 |

The above table shows the maximum value of variable x2 which can be used to satisfy the inequality in case of packaging department. Sheet variable is the one which has values satisfying the inequality automatically. There is no effect on the profit maximizing output when we change the value of a variable in the constraint because the conditions are fulfilled by a variety of values within the favourable region.

The above graphs are developed by using the various inequalities provided and developed in the present scenario. The feasible region will be the area which is below all the graphs developed. Any point in this region will be attainable and feasible for the company. One issue that is shown by graphical method is that there is no common solution to the whole set of equations and we have only one point in graph where three inequalities converge otherwise there are only two inequalities converging at any given point.

From the above analysis, we can conclude that linear programming techniques can be used to maximize or minimize certain given functions which pertain to profit or cost. There are many assumptions associated with the linear programming usage. The current example is related to profit maximization. We have changed the values of both variables to assess the impact on the profit function. One important aspect of this analysis is that there will be no change in the number of units required for maximizing the profit which means that the business will be indifferent between any point within the given constraints. Managers can consider the relative costs of resources as well as the revenues associated to them. Using these data will mean that there will be a much complete analysis of overall situation for the business because the profit function will be derived from the difference of revenue and expenses or cost.

# **References**

Denardo, E. V. (2011). *Linear programming and generalizations.* London: Springer.

G.V.Shenoy. (1998). *Linear Programming: Methids and applications.* Pune: New Age international publishers.